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Quantum Logic and Physics

George Svetlichny[1]

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Quantum logic can potentially provide important new insights into physics in at least two ways. By exploring the relations between quantum structure and space-time structure one clarifies the origin of the Hilbert space formalism, and by examining highly mixed states in modal models, one can gain insight into the quantum-classical transition necessary to resolve the measurement paradoxes.

1. INTRODUCTION

Quantum logic has its roots in physical theories and was created to provide basic insights into the otherwise puzzling world of quantum mechanics. In spite of this, physics and quantum logic have gone their separate ways, as though up to now each one had little to say to the other. There are indications though that this situation may soon change.

What is becoming progressively clearer is that the currently accepted foundations of modern physics—quantum theory, Lorentzian space-time, indeterminism, and atomism—form a rigid theoretical framework that is, however, structurally unstable in that certain deviations, no matter how small, must necessarily force a total revision of this basis. Quantum logic provides some of the mathematical tools necessary to explore this rigid structure.

The instability referred to is due to the existence of EPR-type long-distance quantum correlations. These, along with the hypothesis of the impossibility of superluminal (faster than light) communication (imposed by Lorentz covariance to avoid causal paradoxes), have the effect of globalizing gross features of the quantum formalism, making them necessarily true universally. There are logical connections between Lorentz covariance, the second law of thermodynamics, and the impossibility of superluminal communication. These connections lay bare what is at stake if the quantum

[1]Departamento de Matemática, Pontifícia Universidade Católica, Rio de Janeiro, Brazil.

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formalism were to fail and the profound revisions that this would cause in the whole framework of modern physics. Recent experiments on macroscopic quantum tunneling suggest that the quantum–classical distinction is basically thermodynamic, providing another opportunity for quantum logic to clarify the underlying concepts.

2. QUANTUM LOGICS AND SPACE-TIME STRUCTURE

Long-distance correlations were used by Einstein, Podolsky, and Rosen (1935) to argue the incompleteness of quantum mechanics. They were subsequently seen as providing evidence for a type of action at a distance though of a very subtle kind. Many wondered if they can be used to propagate signals at a velocity faster than light. The first rebuttal to this idea came from Eberhard (1977, 1978), Ghirardi and Weber (1979), and Ghirardi *et al.* (1980), who showed that the statistics of EPR correlations cannot carry a signal. In contraposition, Herbert (1982) presented schemes that made use of the individual behavior of photons, relying on a "photon duplicator" using stimulated emission. This proposal was rebutted by Dieks (1982), Milonni and Hardies (1982), and Wootters and Zureck (1982), who showed that stimulated emission does not work the way required for a "photon duplicator" and that the very linearity of Hilbert-space quantum mechanics precludes any such device.

Thus, in spite of long-distance correlations, the usual quantum mechanical formalism does not allow superluminal communication. It is remarkable though that to some extent the converse also seems to be true, at least locally, that is, no small deviations from the usual Hilbert-space quantum logic is possible.

In 1989 the author proved the following result (Svetlichny, 1989*a*):

> Assuming the usual Hilbert-space (of dimension three or greater) quantum formalism, including the projection postulate, in relation to one arm of an EPR apparatus, and assuming the impossibility of superluminal communication, then at the distant other arm (1) the mean value of observables must be given by the usual quantum mechanical expectation value formula $\langle A\rangle = \mathrm{Tr}(\rho A)$, (2) density-matrix state transformers must be linear, and (3) vector state transformers whose range contains more than one ray must be either linear or antilinear.

The first result follows from Gleason's (1957) theorem. The second result follows from the first by a simple argument, and the third follows from the second by a generalization of Wigner's theorem (Svetlichny, 1989*b*).

This result shows the rigidity of the Hilbert space structure once the impossibility of superluminal communication is assumed. In particular, no nonlinearity in the evolution of the state wave function can be admitted. Nonlinear evolutions have been proposed at various times as a way to avoid quantum paradoxes. Any such proposal, we see now, necessarily introduces superluminal communication. Recently Weinberg (1989*a*,*b*) proposed a nonlinear extension of quantum mechanics whose effects could in principle be seen in spectral line broadening. Any such effect could be immediately used to set up a superluminal communication device. Gisin (1990) provided a detailed description of such a device for the Weinberg theory and has independently come to conclusions similar to the ones presented here.

The above result is of course local in that it shows that once Hilbert-space quantum mechanics is true for a large enough portion of physical phenomena, then it must be universal if superluminal communication is to be ruled out. The global question is:

What constraints does the impossibility of superluminal communication impose upon a physical quantum logic, and does it in particular imply that the logic is Hilbertable?

It seems plausible that the answer is affirmative and that axioms necessary do derive Hilbert space structure, such as Piron's (1976) covering axiom and Ludwig's (1983) axiom on the increase of sensitivity of effects, could be replaced by a "no signal" condition barring superluminal communication. This would provide for the first time a physical reason for Hilbert space and constitute a profound contribution of quantum logic to physics.

3. QUANTUM LOGIC AND THERMODYNAMICS

The "no signal" hypothesis is superficially similar to thermodynamic assertions such as the second law. One can in fact establish a logical link between the two (Svetlichny, 1990*a*). If the mechanism behind superluminal communication satisfies Lorentz covariance, then, as is well known, one can send a signal to one's past. This of course causes serious causal paradoxes and is the main reason for assuming that superluminal communication is impossible. If one however pursues the consequences of such retrograde communication, then one finds that using such signals within closed thermodynamic systems, one can have information about future fluctuations and thereby use it to advantage to extract work from them. This would violate the second law of thermodynamics unless the signaling process itself creates sufficient entropy. What one concludes thus is that Lorentz covariance along with the second law of thermodynamics implies the impossibility of nonentropic superluminal communication. Combining this with the previous result, we can see the consequences of a breakdown of the quantum formalism. If experimental results cannot be explained by the usual Hilbert space expectation value formula, then superluminal communication is possible. If this communication channel is nonentropic, then one must reject either Lorentz covariance or the second law of thermodynamics. In any case, Lorentz covariance is put to question because of causal paradoxes.

One has further reasons to believe that thermodynamic considerations are relevant to quantum foundations. Recent experiments on macroscopic quantum tunneling (Clarke *et al.*, 1988) in Josephson junction devices provide striking evidence that macroscopic systems (superconducting currents) can be made to act quantum mechanically and enter into superpositions of macroscopically distinguishable states (different potential values across the junction). Not only does this provide laboratory analogs of some of the quantum measurement problems, such as the one known as the Schrödinger's cat paradox, but also forces us to revise our notions of the quantum–classical distinction. We have been accustomed to think that this distinction coincides with the microscopic–macroscopic one, but the experimental evidence implies that this is not so. In the author's view what distinguishes quantum systems from classical ones is not the number of particles, but probably some thermodynamic quantity such as the entropy. Classical systems are generally taken to be in highly mixed states (described by density matrices). A mixed state under the usual view is thought to be one that describes an ensemble for which each individual member can be construed to be in some pure state, and these pure states appear in probabilistic proportions that define the mixture. In our view this is a correct description only in certain cases, such as the mixture that obtains on one arm of an EPR-type apparatus when a measurement is performed on the other arm. In contrast, classical systems would be intrinsically mixed and could never even in principle exist in a pure quantum state. This would resolve various paradoxes of quantum measurement, but one lacks a dynamical picture of the process. The full physical theory, subjected to the no-signal hypothesis, would have to be expressible as a linear (nonunitary and possibly stochastic) dynamics of the density matrix that would not only give account of the usual quantum mechanics and resolve the measurement paradoxes, but also be consistent with macroscopic quantum tunneling. This theory may take some time to discover, but one can already explore some of the concepts involved. Any extension of quantum mechanics can be construed as a type of hidden variable theory. One can see within such theories some of the thermodynamic properties that are needed to express the new dynamics.

In 1986 the author showed (Svetlichny, 1986) that finite quantum logics can be interpreted within modal logic so as to reproduce the newly introduced "quantum supports" of Foulis *et al.* (1983) (FPR). This construction (Svetlichny, 1990*b*) was extended to show that the modal models in fact provide what in other contexts of quantum foundations are called contextual hidden-variable theories. Within this interpretation the "quantum states" are represented by the FPR supports, while the ultimate ontological states (hidden variables) are represented by minimal transversals of the block hypergraph of the quantum logic. Pure states correspond to irredundant supports and mixed states to unions of these. One discovers through computer calculations that in some logics there are minimal transversals that are not contained in any irredundant support and so these ontological states can only manifest themselves as mixed states. This is precisely the situation that is needed to provide a transition from quantum to classical along some thermodynamic scale. Computer studies are underway to explore these situations.

One sees therefore from all these considerations that quantum logic still holds out the promise of providing new fundamental insights into physical theories.

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REFERENCES

Clarke, J. C., Cleland, A. N., Devoret, M. H., Esteve, D., and Martins, J. M. (1988). *Science*, **239**, 992.

Dieks, D. (1982). *Physics Letters*, **92A**, 271.

Eberhard, P. H. (1977). *Nuovo Cimento B*, **38**, 75.

Eberhard, P. H. (1978). *Nuovo Cimento B*, **46**, 392.

Einstein, A., Podolsky, B., and Rosen, N. (1935). *Physical Review*, **47**, 777.

Foulis, D., Piron, C., and Randall, C. (1983). *Foundations of Physics*, **13**, 813.

Ghirardi, G. C., and Weber, T. (1979). *Lettere Nuovo Cimento*, **26**, 599.

Ghirardi, G. C., Rimini, A., and Weber, T. (1980). *Lettere Nuovo Cimento*, **27**, 293.

Gisin, N. (1990). *Physics Letters A*, **143**, 1.

Gleason, A. M. (1957). *Journal of Mathematics and Mechanics*, **6**, 885 [reprinted in C. A. Hooker, ed., *The Logico-Algebraic Approach to Quantum Mechanics*, Vol. I, D. Reidel, Dordrecht, 1975].

Herbert, N. (1982). *Foundations of Physics*, **12**, 1171.

Ludwig, G. (1983). *Foundations of Quantum Mechanics*, Springer-Verlag, New York.

Milonni, P. W., and Hardies, M. L. (1982). *Physics Letters*, **92A**, 321.

Piron, C. (1976). *Foundations of Quantum Physics*, Benjamin, London.

Svetlichny, G. (1986). *Foundations of Physics*, **16**, 1285.

Svetlichny, G. (1989*a*). The quantum formalism and superluminal communication, Preprint, Departamento de Matemática, Pontifícia Universidade Católica, Rio de Janeiro, Brazil.

Svetlichny, G. (1989*b*). On linear transforms of rank-one positive operators, Preprint, Departamento de Matemática, Pontificia Universidade Católica, Rio de Janeiro, Brazil.

Svetlichny, G. (1990*a*). Lorentz covariance, the second law of thermodynamics, superluminal communication, and quantum mechanics, Preprint, Departamento de Matemática, Pontifícia Universidade Católica, Rio de Janeiro, Brazil.

Svetlichny, G. (1990*b*). *Foundations of Physics*, **20**, 635.

Weinberg, S. (1989*a*). *Physical Review Letters*, **62**, 485.

Weinberg, S. (1989*b*). *Annals of Physics*, **194**, 336.

Wootters, W. K., and Zurek, W. H. (1982). *Nature*, **299**, 802.